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Acceleration levels of dropped objects

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Introduction

This paper is intended to provide an overview of drop shock testing, which is defined as the acceleration experienced by an object dropped onto a hard surface. The practical application of this work is to characterize the acceleration levels generated when an accelerometer is dropped onto a rigid surface. Similar dynamic responses can be inferred to the object on which the accelerometer is mounted.

An accelerometer is a particularly convenient object to evaluate, since it may directly characterize the amplitude and pulse width generated from an impact. Many accelerometers have maximum acceleration ratings which are sufficient to measure high acceleration levels in a wide variety of applications. It should be realized that in almost every case, ratings may be exceeded by seemingly benign handling shocks.

Summary

It has been demonstrated that when dropped from a height of one meter, objects may impact rigid surfaces with very large accelerations. Relatively speaking, lower mass objects tend to result in greater accelerations. As an example, an accelerometer, with a normal mass of only 10 grams, may easily be subjected to 30 000 g when dropped from one meter onto a rigid steel plate. If the interface on impact is flat-to-flat, the level may exceed 80 000 g.

The amount of acceleration imparted to a dropped object is proportional to the square root of the drop height, and the inverse of the pulse width.

The drop shock

Consider an object dropped onto a horizontal surface. For the purpose of this evaluation, the object starts at rest, and free-falls to impact a rigid surface after traveling a distance, d_1, as shown in Figure 1. After impact, the object rebounds upward to some height d_2.

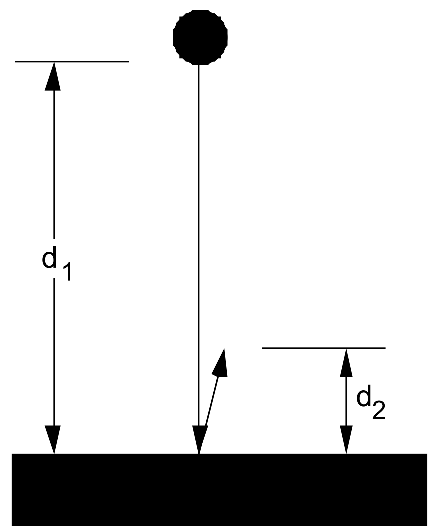


Figure 1

An object in free-fall experiences constant acceleration due to gravity, and the velocity increases in the downward (negative) direction as

$$v(t) = g\ t \qquad (1)$$

The distance traveled is

$$d(t) = \frac{1}{2} g\ t^2 \qquad (2)$$

These two equations may be combined to determine the velocity of the object immediately prior to impact.

$$v_1 = -\sqrt{2\ g\ d_1} \qquad (3)$$

Likewise, the upward velocity immediately after impact is

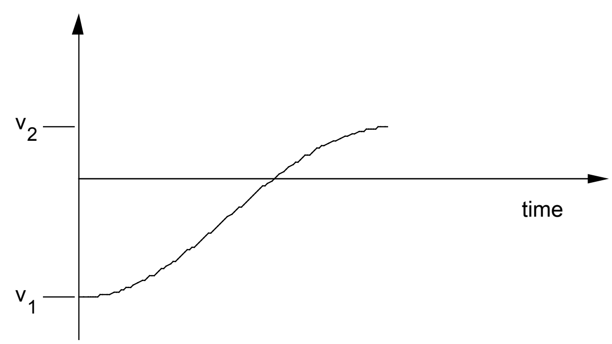
$$v_2 = \sqrt{2\ g\ d_2} \qquad (4)$$

Immediately before impact, the object is still in free-fall. If the object were a piezoelectric accelerometer, it would have zero output. The same is true immediately after impact. During impact, the object must stop its downward course, and reverse its direction. This is where the high level acceleration occurs. Acceleration is, by definition, the rate of change of velocity. The faster the object changes from downward velocity to upward velocity, the greater the acceleration.

(5)

Paraphrasing Newton, an object in motion tends to remain in motion ... and a more massive object requires more force to accelerate it. This implies that, in general, more massive objects take more time to rebound. Consider two objects of equal shape and rigidity, one more massive than the other. Both fall and rebound the same distance, so the change of velocity is equal in both cases. However, the more massive object takes more time to rebound (and has a longer pulse width), so Δt is greater, resulting in less acceleration as shown by

The velocity is the integral of the acceleration, plus an integration constant. Integrating the half sine results in a 'half cosine', and would look something like that shown in Figure 3, below. Keep in mind that the curve can be shifted vertically to match the boundary conditions of the drop (thanks to the integration constant).



To set the boundary conditions of the velocity curve, first realize that v1 is simply the velocity immediately before impact.

(6)

And v2 is the velocity immediately after impact

(7)

The curve of figure 3 with the above two boundary conditions is described by

$$v(t) = \cos\left(\frac{\pi t}{T}\right)\left(\frac{v_1 - v_2}{2}\right) + \frac{v_1 + v_2}{2} \qquad (8)$$

where T is the pulse width of the impact. Taking the derivative of the velocity function yields the acceleration function

$$a(t) = \frac{d}{dt} v(t) = \frac{d}{dt}\left(\cos\left(\frac{\pi t}{T}\right)\left(\frac{v_1 - v_2}{2}\right) + \frac{v_1 + v_2}{2}\right) \qquad (9)$$

$$a(t) = -\frac{\pi}{T}\sin\left(\frac{\pi t}{T}\right)\left(\frac{v_1 - v_2}{2}\right) \qquad (10)$$

such that

$$a(0) = 0 \qquad (11)$$

$$a\left(\frac{T}{2}\right) = \frac{\pi}{T}\left(\frac{v_2 - v_1}{2}\right) \qquad (12)$$

$$a(T) = 0 \qquad (13)$$

all which describe the characteristics of a half sine acceleration pulse. The peak acceleration occurs at time T/2, as seen in equation (12). Substituting the velocity variables,

$$a_{peak} = a\left(\frac{T}{2}\right) = \frac{\pi}{T}\left(\frac{v_2 - v_1}{2}\right) \qquad (14)$$

$$a_{peak} = \frac{\pi}{T}\left(\frac{\sqrt{2\,g\,d_2} - \left(-\sqrt{2\,g\,d_1}\right)}{2}\right) \qquad (15)$$

$$a_{peak} = \frac{\pi}{T}\sqrt{\frac{g}{2}}\left(\sqrt{d_2} + \sqrt{d_1}\right) \qquad (16)$$

From equation (16) it can be seen that the peak acceleration in a drop shock is primarily dependent on two things:

1. **The inverse of the pulse width. Shorter pulse widths result in higher accelerations.**

2. **The square root of the drop and rebound distance. Assuming minimal rebound, doubling the drop height results in a 41% higher shock.**

Here are some simplified approximations:

$$g_{peak} = .71\,\frac{\sqrt{d_2} + \sqrt{d_1}}{T} \quad \text{for } d_1, d_2 \text{ in m : T in s} \qquad (17)$$

$$g_{peak} = .39\,\frac{\sqrt{d_2} + \sqrt{d_1}}{T} \quad \text{for } d_1, d_2 \text{ in ft : T in s} \qquad (18)$$

The pulse width 'T' is dependent on the shape and material properties of the dropped object and the impact surface.

Simple prediction model of drop shocks

For generalization purposes, consider the dropped object a solid mass of some material. When the dropped object impacts a rigid surface, the object deforms (compresses), and may rebound back upwards. For simplicity, assume no rebound.

The dropped object has mass m, and modulus E. The distance the object compresses is x. From basic formulae

$$F = m\,a \qquad (19)$$

$$F = k\,x \qquad (20)$$

we can find the acceleration imparted at the maximum compression distance, x. This is what we want.

$$a = \frac{k\,x}{m} \qquad (21)$$

During the shock,

$$v = \sqrt{2\ a\ x} \qquad (22)$$

$$x = \frac{v^2}{2\ a} \qquad (23)$$

Substituting (23) into (21) yields

$$x = \frac{k\ \frac{v^2}{2\ a}}{m} \qquad (24)$$

$$a^2 = \frac{k\ v^2}{2\ m} \qquad (25)$$

The spring constant of a bulk spring can be expressed in terms of the modulus, the area being compressed and the height.

$$k = \frac{E\ A}{h} \qquad (26)$$

Substituting (22) and (26) into (25) yields

$$a^2 = \frac{\frac{E\ A}{h}\left(2\ g\ d_1\right)}{2\ m} \qquad (27)$$

which simplifies to

$$a = \sqrt{\frac{E\ A\ g\ d_1}{h\ m}} \qquad (28)$$

From this basic equation, a number of conclusions can be deduced.

1) Greater acceleration levels will be achieved when the surface area, A, is increased. Specifically, if an object is dropped onto its most flat surface (flat-on-flat impacts) the acceleration is the greatest. Conversely, if dropped onto a sharp corner, the acceleration will be minimized.

2) Greater acceleration levels will be achieved with a higher drop (this should be obvious).

3) Greater acceleration levels will be achieved with a lower mass object.

4) Greater acceleration levels will be achieved with a stiffer object (higher modulus).

The following table shows the estimated peak acceleration levels at various drop height and pulse width.

pulse width msec ↓ / inch →	6	12	24	36	48	60
0.010	28000	39000	55000	68000	78000	87000
0.013	21000	30000	42000	52000	60000	67000
0.016	17000	24000	34000	42000	49000	55000
0.020	14000	20000	28000	34000	39000	44000
0.025	11000	16000	22000	27000	31000	35000
0.032	8600	12000	17000	21000	24000	27000
0.040	6900	10000	14000	17000	20000	22000
0.050	5500	7800	11000	14000	16000	17000
0.063	4400	6200	8800	11000	12000	14000
0.079	3500	4900	7000	8600	10000	11000
0.100	2800	3900	5500	6800	7800	8700
0.13	2200	3100	4400	5400	6200	6900
0.16	1700	2500	3500	4300	4900	5500
0.20	1400	2000	2800	3400	3900	4400
0.25	1100	1600	2200	2700	3100	3500
0.32	870	1200	1700	2100	2500	2800
0.40	690	1000	1400	1700	2000	2200
0.50	550	780	1100	1300	1600	1700
0.63	440	620	870	1100	1200	1400
0.79	350	490	690	850	1000	1100
1.0	280	390	550	680	780	870
1.3	220	310	440	540	620	690
1.6	170	250	350	430	490	550
2.0	140	200	280	340	390	440
2.5	110	160	220	270	310	350
3.2	87	120	170	210	250	280
4.0	69	100	140	170	200	220
5.0	55	78	110	130	160	170
6.0	46	65	90	110	130	150
8.0	34	49	69	84	100	110
10	28	39	55	68	78	87
12	23	33	46	56	65	73
16	17	25	35	43	49	55
20	14	20	28	34	39	44
25	11	16	22	27	31	35
32	9	12	17	21	25	28
40	7	10	14	17	20	22
50	6	8	11	13	16	17
63	4	6	9	11	12	14
79	4	5	7	9	10	11
100	3	4	6	7	8	9

Measuring a dropped object

Due to the high level of acceleration generated during the drop impact, selecting the right accelerometer and signal conditioner is critical to a good measurement. A few areas need special attention:

1. **Size of the object to be Measured**-If the selected accelerometer is too big, its presence may alter the response characteristics of the object. As a rule of thumb, the accelerometer should not be more than 10th of the weight of the measured object.

2. **Accelerometer mounting**-Although the mounting method is practically dictated by the selected accelerometer, it is important to keep in mind that screw mounting is preferred in shock environments.

3. **Type of measurement**-Are you trying to measure the rigid body motions of the dropped object (i.e., how much does it bounce), or are you interested in the peak acceleration responses of the object? If rigid body motions are desired, the measurement instrumentation chain must have DC response in order to minimize zero-offset errors.

4. **Dynamic characteristics of accelerometer**-The accelerometer must have the proper frequency response and dynamic range. It is recommended to use the table above as a guide and apply a safety margin of 2 in reaching the desired specifications. For more information on selecting the right accelerometer, refer to TP291 Accelerometer Selection Based on Application, available for download at www.endevco.com.

5. **Transfer characteristics of amplifier**-Given the high level and wide bandwidth signal output from the accelerometer, the amplifier must handle the signals without producing spurious distortion. Frequency and overload characteristics of the amplifier must be well understood before making a selection.

6. **Frequency range and Resolution of analog-to-digital conversion**-Attention must be paid in selecting the proper bit and sampling rates in A/D converter. Sampling rate must be high enough to cover the bandwidth of the acceleration signals, which can sometime exceed 200 kHz. Sufficient bit resolution is critical for deriving rigid body motion information from an accelerometer. Depending on the desired accuracy, low bit rate can result in gross integration errors during signal post processing.